Year 12 Further Maths Summer Work (Task 1 of 2)

You have two tasks to complete over summer to prepare you for your Further Maths A level course. You must bring both of these tasks to your first **Maths** (not Further Maths) class in September.

Task 1:

Complete the problem solving questions below. They are all challenges based on GCSE maths theory. You should know everything you need to know already but they will require some problem solving skills.

1.

Challenge

Expand and simplify $(x + y)^4$.

2.

Challenge

Write $4x^4 - 13x^2 + 9$ as the product of four linear factors.

3.

Challenge

a Simplify $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$.

b Hence show that $\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \ldots + \frac{1}{\sqrt{24} + \sqrt{25}} = 4$

4.

Challenge

Given that x is positive, solve the equation

$$\frac{1}{x}+\frac{1}{x+2}=\frac{28}{195}$$

5.

Challenge

a Prove that, if the values of a and c are given and non-zero, it is always possible to choose a value of b so that $f(x) = ax^2 + bx + c$ has distinct real roots.

b Is it always possible to choose a value of b so that $f(x)$ has equal roots? Explain your answer.

6.

Challenge

Accident investigators are studying the stopping distance of a particular car.
When the car is travelling at 20 mph, its stopping distance is 6 feet.
When the car is travelling at 30 mph, its stopping distance is 14 feet.
When the car is travelling at 40 mph, its stopping distance is 24 feet.

The investigators suggest that the stopping distance in feet, d, is a quadratic function of the speed in miles per hour, s.

a Given that $d(s) = as^2 + bs + c$, find the values of the constants a, b and c.

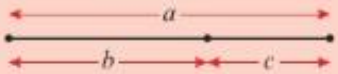
b At an accident scene a car has left behind a skid that is 20 feet long.
Use your model to calculate the speed that this car was going at before the accident.

Hint Start by setting up three simultaneous equations. Combine two different pairs of equations to eliminate c. Use the results to find the values of a and b first.

7.

Challenge

a The ratio of the lengths $a : b$ in this line is the same as the ratio of the lengths $b : c$.



Show that this ratio is $\frac{1+\sqrt{5}}{2} : 1$.

b Show also that the infinite square root

$$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}}} = \frac{1+\sqrt{5}}{2}$$

8.

Challenge

a Show that the solutions to the equation $ax^2 + 2bx + c = 0$ are given by $x = -\frac{b}{a} \pm \sqrt{\frac{b^2 - ac}{a^2}}$.

b Hence, or otherwise, show that the solutions to the equation $ax^2 + bx + c = 0$ can be written as $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.